“Robust” Design and Optimization Under Uncertainty

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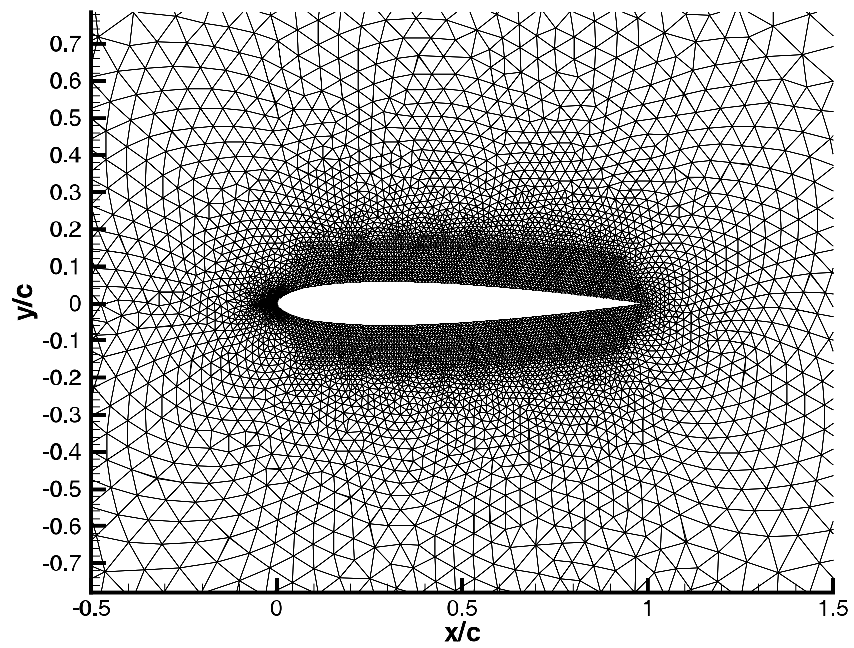
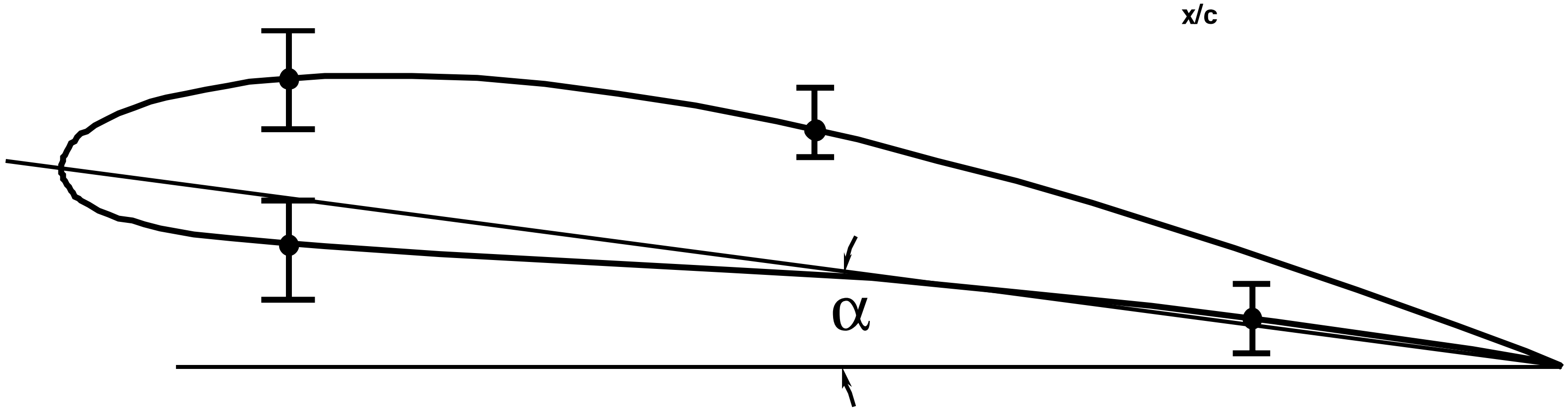
Airfoil Geometry Optimization

- Find optimal airfoil geometry, which results in minimum drag c_d over a range of free flow Mach numbers M while maintaining a given target lift c_l^*. The NACA-0012 is the baseline design.
- For this example we assume that the Mach number $M \in [0.7, 0.8]$. The Mach number cannot fall outside this interval.
- We solve the inviscid Euler equations using NASA's FUN2D code, which computes accurate derivatives. Far field boundary at 50 chord lengths.

Design Variables in FUN-2D

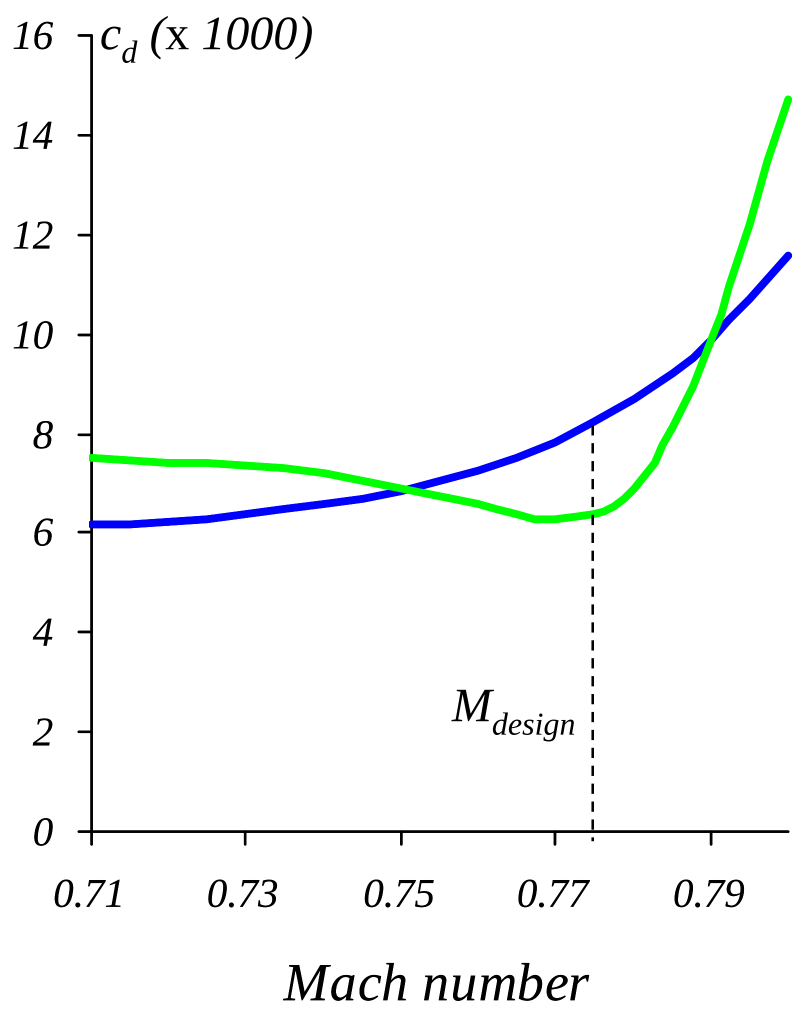
- Design vector *d*: angle of attack and 20 box-constrained *y*-coordinates of the control points for the airfoil spline



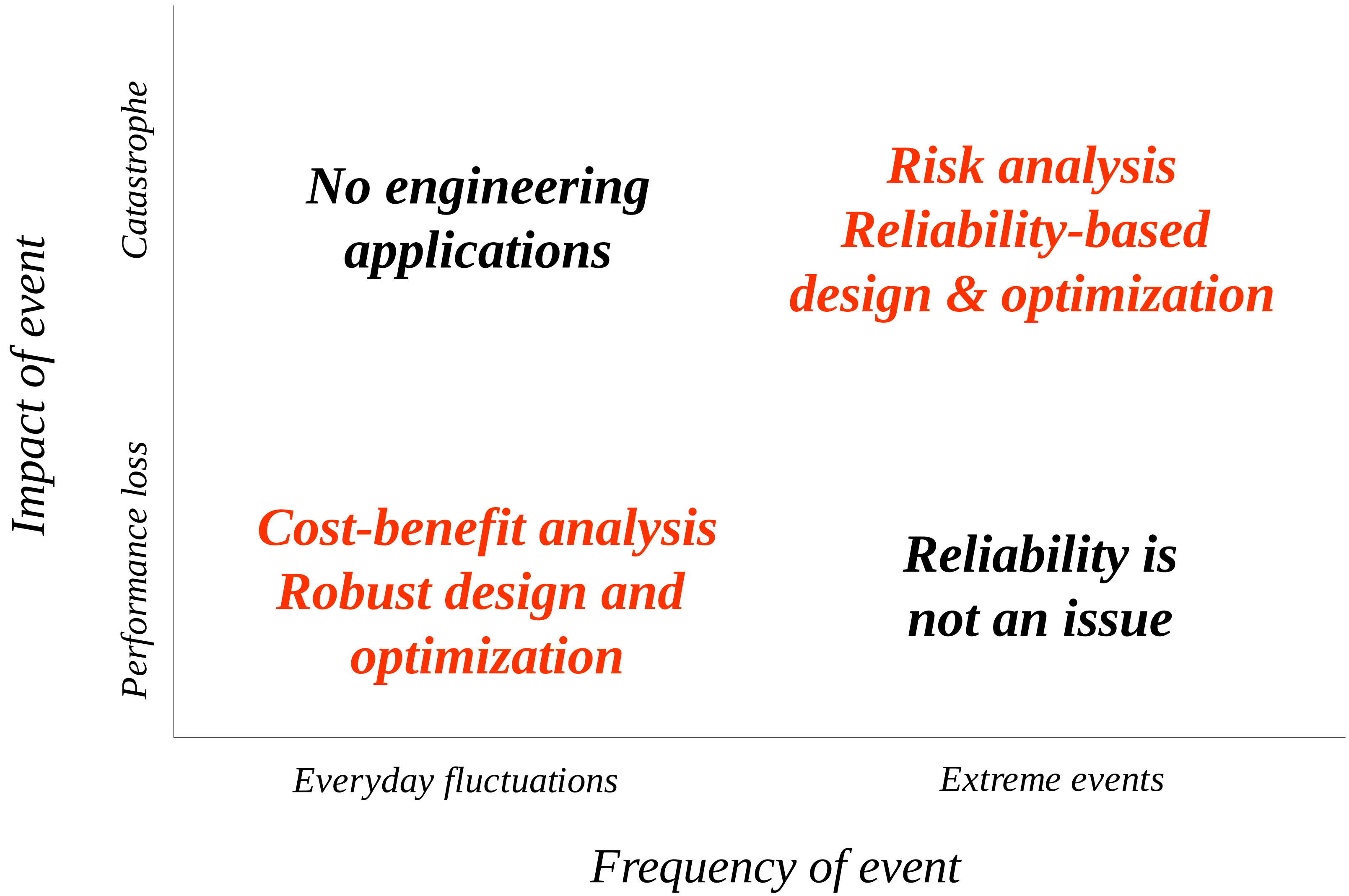


Research Objective

- Inherent variability associated with many design parameters.
- With conventional optimization techniques:
  - * Impact of such fluctuations is unknown.
  - * The performance away from the design point can be worse for the optimized design than for the baseline design.



Reliability Problem Classification



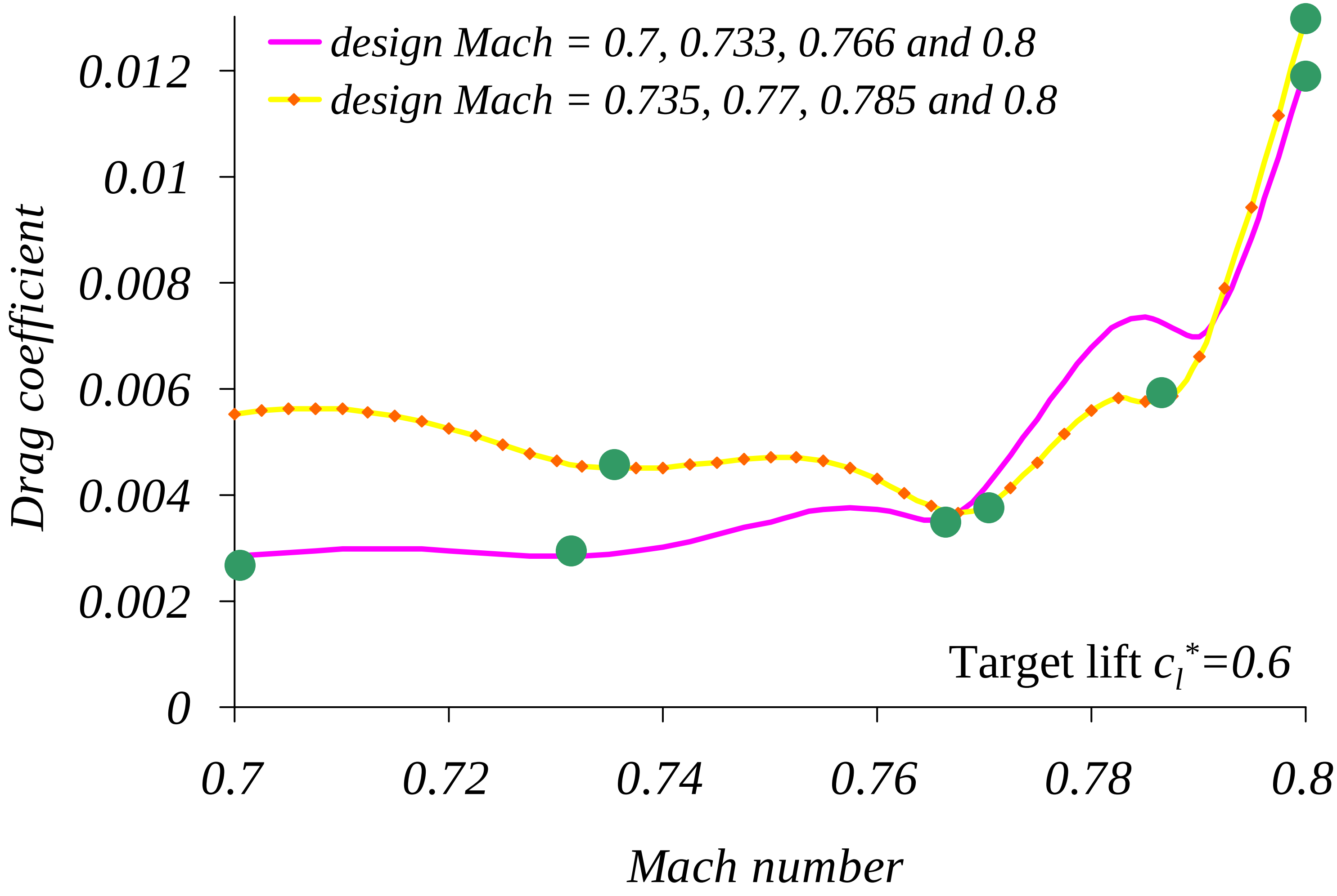
Multi-Point Optimization

- The design vector *d* (geometry and angle of attack) is the only variable in the objective
- Consider multiple design conditions at selected values of the free flow Mach number
- Objective function is a weighted average of all these design conditions

$$\begin{cases} \min\limits_{d \in D} \quad \sum_{i=1}^{n} w_i c_d(d, M_i) \\ \text{subject to} \quad c_l(d, M_i) \geq c_l^* \quad \text{for} \quad i = 1, n \end{cases}$$

Problems with Four-Point Opt.

Choice of design conditions affects performance



Problems of Conventional Methods

Drawbacks are two-fold, there is no clear answer to the following two questions:

- Which operating conditions should be included in the objective function?
- What are the weights associated with each operating condition?

Possible Goals for Robust Design

- Maximize worst-case performance *(non-probabilistic)*
- Maximize the consistent improvement of an existing design over the entire range *(non-probabilistic)*
- Minimize the performance fluctuation over the entire range *(probabilistic)*
- Maximize the overall expected value of the performance *(probabilistic)*

Each method typically results in a different design!

Profile Optimization

- Based on an adaptive-weight minimax problem formulation.

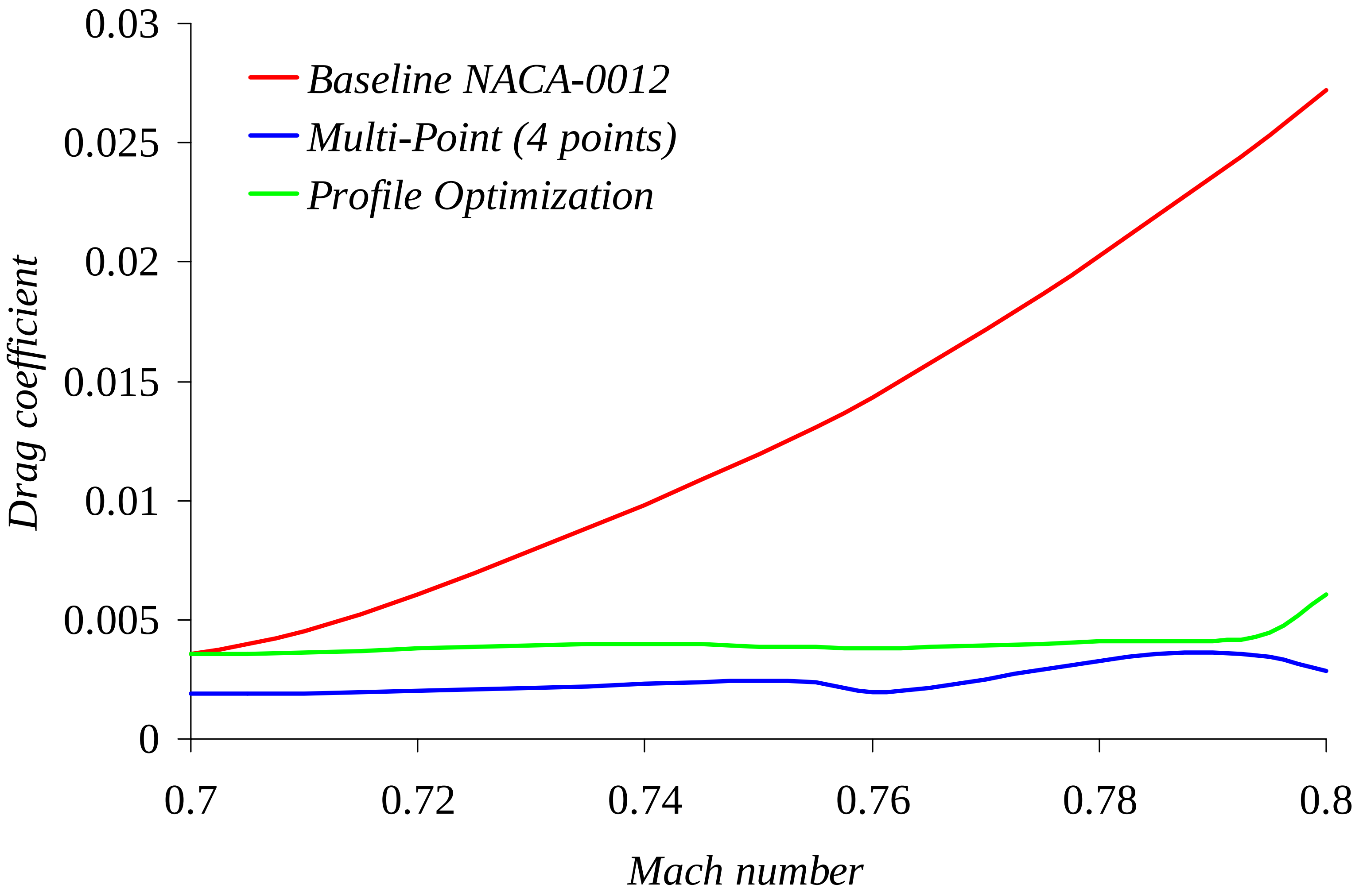
$$\min_{D} \max_{M_{\min} \leq M \leq M_{\max}} \rho(M) c_d(d, M)$$

- For a particular sequence of weights ρ*(M)*, it can be shown to revert to the multipoint optimization algorithm.

Summary of Profile Optimization

- Method requires a priori selection of design conditions.
- In each step a reduction of the drag at each of the design Mach numbers is achieved.
- The algorithm terminates when it is impossible to achieve a consistent drag reduction at all design Mach numbers.

Comparison with Multipoint



Stochastic Optimization

- Modify the objective to directly incorporate the effects of model uncertainties on the design performance
- A full Expected Value Optimization can be accelerated using second-order approximate results for the expected value instead of a full integration.

Mathematical Formulation

Minimize the expected value of the drag over the design lifetime:

$$\min_{d\in D} E_M\left[c_d(d,M)\right] = \min_{d\in D} \int_M c_d(d,M)\, f_M(M)\, dM$$

c_d is drag function

d is design vector (geometry, angle of attack)

M is uncertain parameter (Mach number)

f_M is probability density function of Mach number

SOSM Approximation

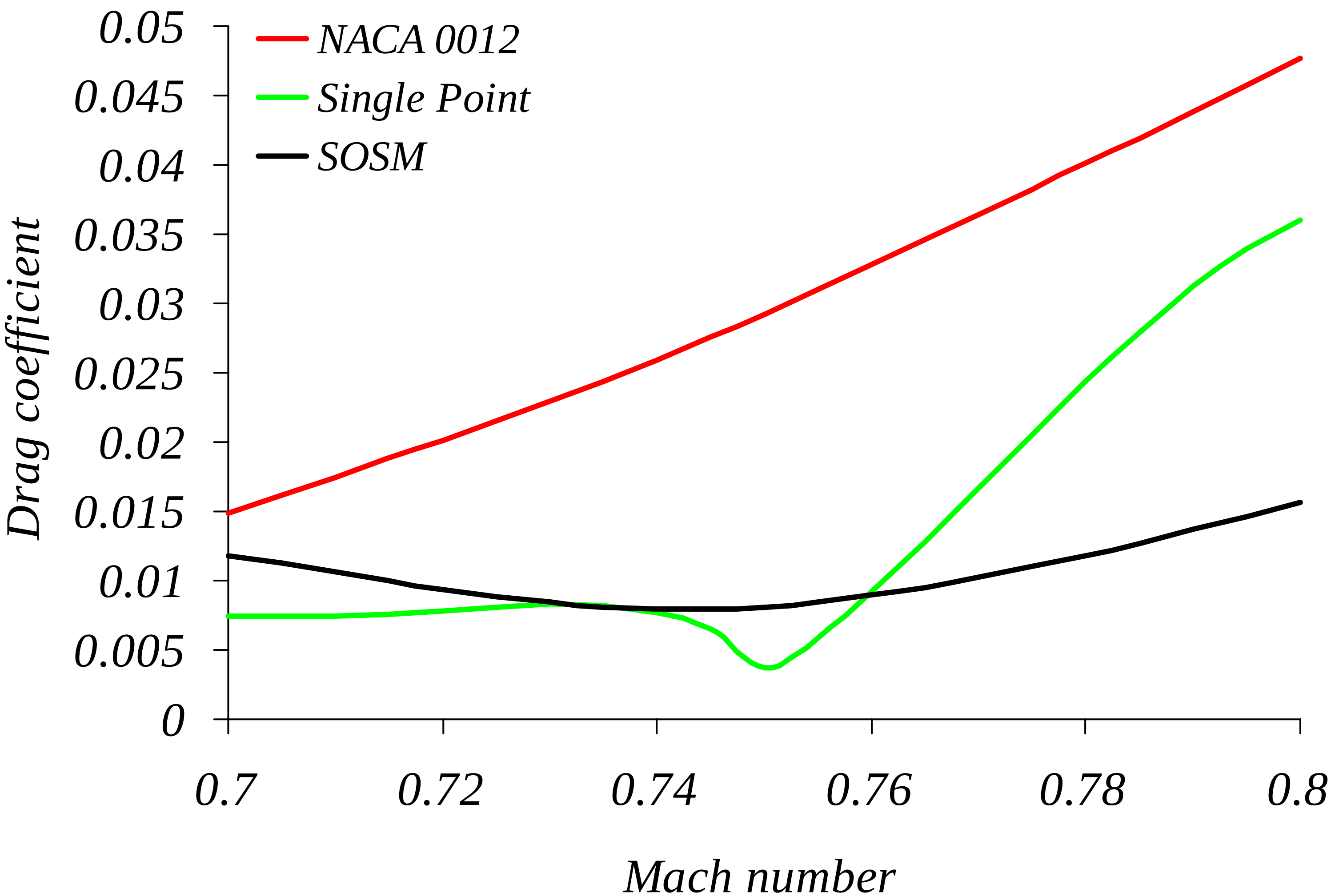
Approximate objective by second-order Taylor series expansion about the mean value of *M*, and evaluate the expectation integral analytically.

$$\min_{d \in D} \int_M c_d(d, M)\, f_M(M)\, dM \cong$$

$$\min_{d \in D} \left[c_d(d, \overline{M}) + \tfrac{1}{2} Var(M) \left. \frac{\partial^2 c_d}{\partial M^2} \right|_{M = \overline{M}} \right]$$

$$subject\ to: \quad c_l \geq c_l^*$$

Comparison with Single Point Opt.



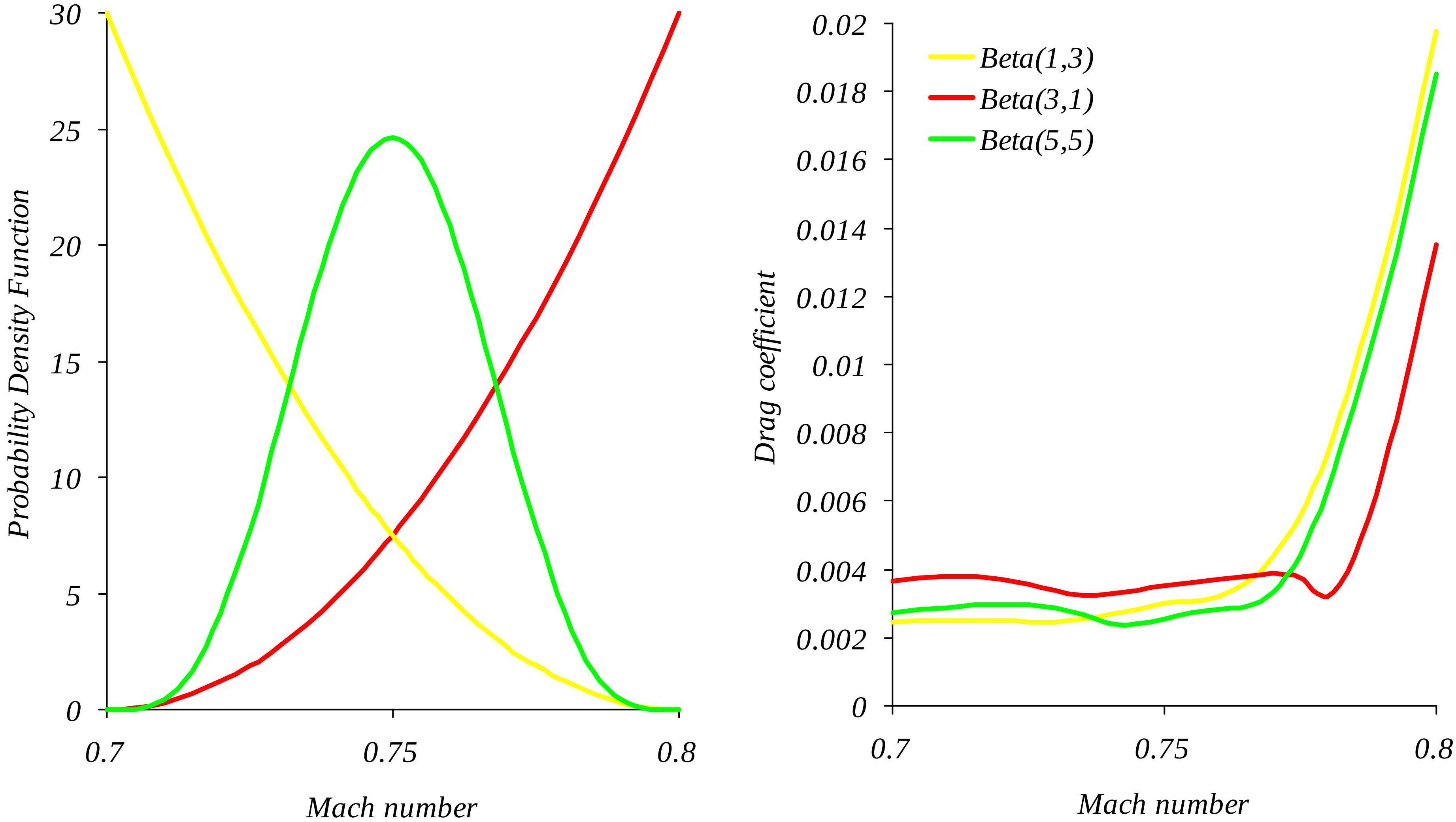
Comparison with Single Point Opt.

- Second-Order information represents curvature of c_d-M curve.
- The weighting between drag and design point and curvature depends on the variance of the Mach number.
- With SOSM method the drag is not reduced quite as much as for single point design but the drag is much less sensitive to variations in the Mach number. The drag trough is avoided, no “over-optimization”.

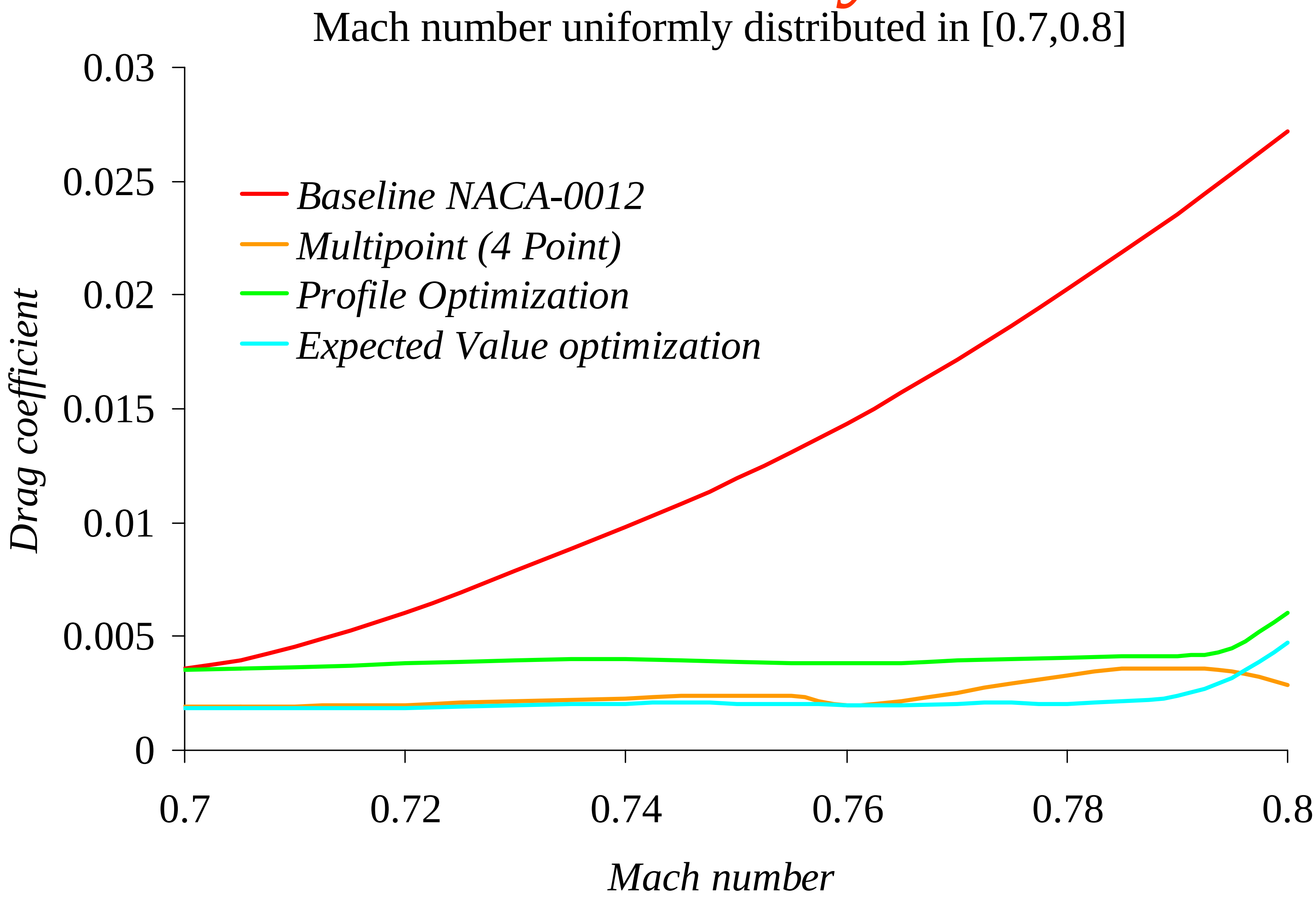
Impact of PDF choice on Profile

- The Expected Value Optimization results automatically reflect the relative importance of each of the Mach numbers. There is no need to arbitrarily select design conditions (i.e. Mach numbers) or weights any longer.
- This is illustrated for 3 Beta distributions: red curve has highest density for high Mach numbers and this results in the lowest drag for high Mach number.

Choice of PDF & Optimal Drag Rise



Summary


Mach number uniformly distributed in [0.7,0.8]
Baseline NACA-0012
Multipoint (4 Point)
Profile Optimization
Expected Value optimization
Drag coefficient
0.03
0.025
0.02
0.015
0.01
0.005
0
0.7
0.72
0.74
0.76
0.78
0.8
Mach number

Conclusions

- Profile optimization: this design cannot be improved unless a trade-off between the performance at different Mach numbers is allowed*
- Expected value optimization: best overall performance of all possible designs for the given Mach number probability distribution

** This was not achieved in our example because the optimizer got stuck in a local minimum*

References

- Luc Huyse and R. Michael Lewis, Aerodynamic shape optimization of two-dimensional airfoils under uncertain operating conditions, NASA/CR-2001-210648, ICASE Report No. 2001-1
- Luc Huyse, Free-form airfoil shape optimization under uncertainty using maximum expected value and second-order second-moment strategies, NASA/CR-2001-211020. ICASE Report No. 2001-18
- Wu Li, Luc Huyse and Sharon Padula, Robust airfoil optimization to achieve consistent drag reduction over a Mach range, NASA/CR-2001-211042, ICASE Report No. 2001-22